FFT Laboratory Experiment 1

Sampling Rate, Frequency Resolution and Spectral Leakage for a Sinusoidal Input Signal

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Purpose:

This simple experiment illustrates the relationship between the effective sampling rate and the resulting frequency resolution for spectral analysis using an FFT. The spectral leakage properties of the rectangular and Hanning windows are also demonstrated. Comparisons can be made between the theoretical results for an FFT analysis of a sinusoid and the experimental results obtained using the FFT module.

Procedure:

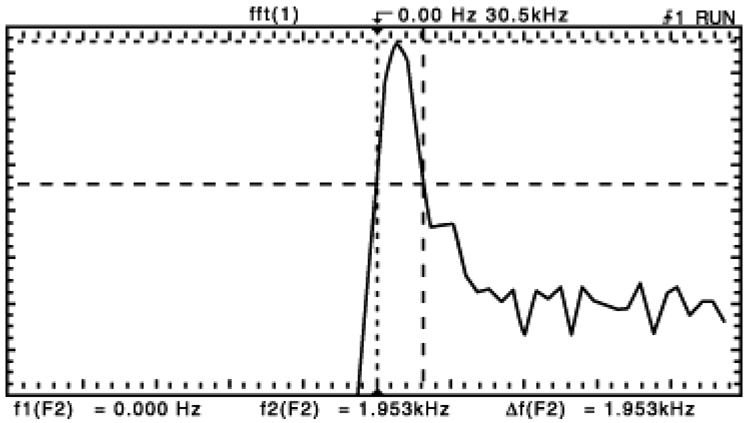
1) Connect a 3.5 V (peak-to-peak) sinusoidal input signal to channel 1 of the oscilloscope. Adjust the frequency of the sinusoid to approximately 1 kHz. Note that the Agilent 54654A Training Kit provides a circuit board capable of producing the desired sinusoid (pin 12).

2) Use *Autoscale* to display the time-domain waveform. Measure the frequency of the sinusoid by first selecting *Time* and then choosing the *Freq* selection from the *Time Measurements* portion of the menu.

3) Next, go to the *Function* menu by depressing the ± key. Under the "Function 2" portion of the menu select *On.* In order to display a realtime vector display of the FFT, select 1 on the front display and then depress the key under the 1 on the screen menu. Adjust the FFT menu settings to the values shown in Figure 2.1 (See the Agilent 54657A and 54658A Measurement Storage Module User Guide page 28 for assistance in adjusting the FFT Menu settings).

4) Use the *Cursors and Find Peaks* automated measurement features to measure the fundamental frequency of the sinusoid. Note that the main lobe width of the Hanning window is roughly 2 kHz (use the *Cursors* menu to measure the width of the main lobe). Next, change to the rectangular window as indicated in Figure 2.2. Note that the width of the main lobe is reduced to roughly 1 kHz. However, the spectral leakage is significantly increased using this window.

   Figure 2.3 shows the "theoretical" 1024 point FFT for a 1 kHz signal using a Hanning window. Recall that the FFT produces frequency samples of the Fourier Transform of the "analog" sampled signal $x_S(t)$. Figure 2.3 shows the Fourier Transform of a 1 kHz sinusoid (1 *V* (RMS) using a 1024 point Hanning window and an effective sampling rate of 500 *kSa / s.* Figure 2.4 shows the Fourier Transform of the same signal using a rectangular window. Note that the Fourier Transforms of the Hanning windowed sinusoid (see Figure 1.6 part (c)), and the rectangular windowed sinusoid (see Figure 1.6 part (b)) have distinctive side lobes which are not readily apparent in Figures 2.3 and 2.4. This is due to the sampling nature of the FFT. The frequency samples are not spaced sufficiently close to graphically depict the individual side lobes. This also, in part, explains the "dancing lines" which appear on the FFT display. As each FFT computation is displayed, there are small deviations in the exact points where the spectrum is sampled. Time-domain sampling jitter and noise also contribute to the "dancing lines" effect.

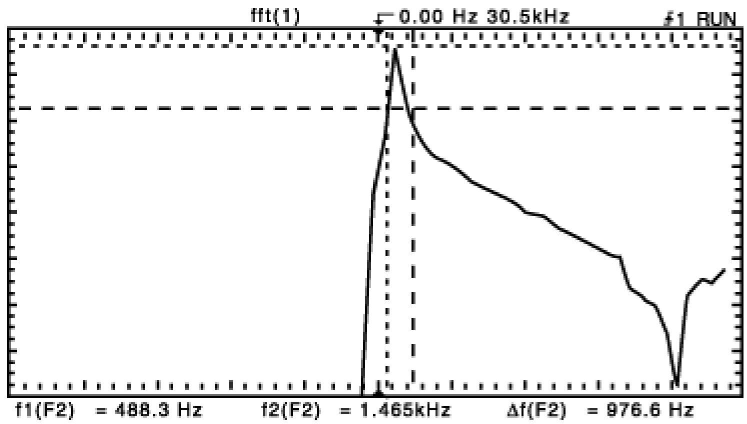


Function 2: Menu		
Effective Sampling Rate	**Units/div**	**Ref Level**
500 kSa/s	**10.00 dB**	**5.00 dBV**

FFT Menu		
Center Freq.	**Freq. Span**	**Window**
0.000 Hz	**30. 52 kHz**	**Hanning**

Figure 2.1

The 1024 DFT (Magnitude) of a 3.5 Volt (peak-to-peak) 1 kHz Sinusoid Sampled at *500 kSa/s* using a Hanning window. The main lobe width is measured by setting the V_1 cursor to the peak of the lobe, and the V_2 cursor to 31 dB below V_1. As shown above, the f_1 and f_2 cursors are then adjusted to the points where the V_2 cursor and the main lobe intersect. As illustrated in Figure 1.6 part *c*, the side lobes associated with the Fourier Transform of the Hanning window are 31 *dB* below the main lobe peak.



Function 2: Menu		
Effective Sampling Rate	Units/div	Ref Level
500 kSa/s	10.00 dB	5.00 dBV
FFT Menu		
Center Freq.	Freq. Span	Window
0.000 Hz	30. 52 kHz	Rectangular

Figure 2.2

The 1024 DFT (Magnitude) of a 3.5 Volt (peak-to-peak) 1 kHz Sinusoid Sampled at 500 *kSa/s* using a rectangular window. The main lobe width is measured by setting the V_1 cursor to the peak of the lobe, and the V_2 cursor to 13 dB below V_1. As shown above, the f_1 and f_2 cursors are then adjusted to the points where the V_2 cursor and the main lobe intersect. As illustrated in Figure 1.6 part (b), the side lobes associated with the Fourier Transform of the rectangular window are 13 dB below the main lobe peak.

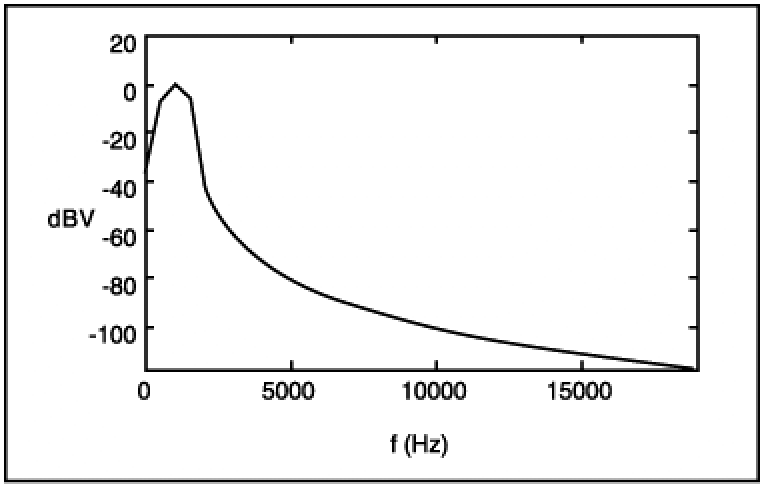


Figure 2.3

The "theoretical" 1024 Point DFT (Magnitude) of a 1 kHz sinusoid (1 V (RMS)) sampled at *500 kSa/s* and using a Hanning window. The side lobes which appear in Figure 1.6 part (c) are "masked" due to the fact that the frequency samples produced by the DFT are spaced too far apart. Zero-padding is often used to improve the graphical presentation of DFTs. (Zero-padding is the process of adding extra zero-value data points to the time waveform before the FFT is processed. See references for details.)

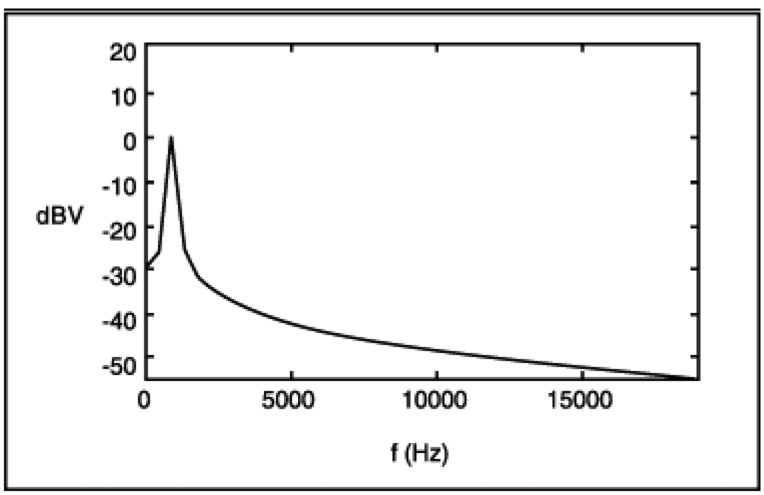
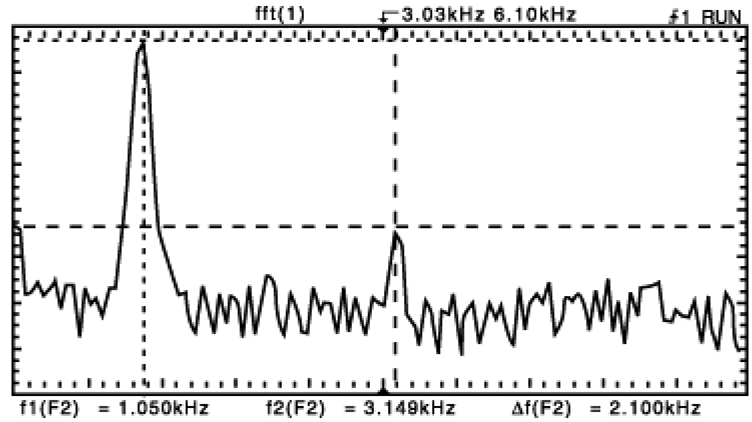


Figure 2.4

Same as figure 2.3, except using a rectangular window.



Function 2: Menu		
Effective Sampling Rate	**Units/div**	**Ref Level**
50 kSa/s	**10.00 dB**	**5.00 dBV**
FFT Menu		
Center Freq.	**Freq. Span**	**Window**
3.027 kHz	**6.104 kHz**	**Hanning**

Figure 2.5

The 1024 point DFT (Magnitude) of a 3.5 V (peak-to-peak) 1 kHz sinusoid sampled at *50 kSa/s* using a Hanning window. The signal board included with the Agilent 54654A Training Kit was used as the input signal. This board intentionally introduces harmonic distortion which is evident in the above figure. One of the vertical cursors indicates the fundamental frequency of the input signal, whereas the other is located at the third harmonic.

5) Use the *Time /Div* control to decrease the effective sampling rate to *50 kSa/s.* Adjust the FFT menu settings to those specified in Figure 2.5. Note that the main lobe width is reduced to roughly 200 Hz for the Hanning window indicating improved frequency resolution. Next, select the rectangular window from the FFT Menu and observe that the mainlobe width for the rectangular window is roughly 100 Hz. Once again, the spectral leakage is quite significant.

6) Repeat the previous step using an effective sampling rate of 10 *kSa / s.* Use Figure 2.6 for assistance in selecting the FFT menu settings. Note that the main-lobe width is roughly 20 Hz for Hanning window and roughly 10 Hz for the rectangular window.

Questions

1. Should the effective sampling rate be increased or decreased in order to improve the frequency resolution of the FFT?

2. Is there a limit on the spectral resolution capabilities of a fixed, 1024 point FFT?

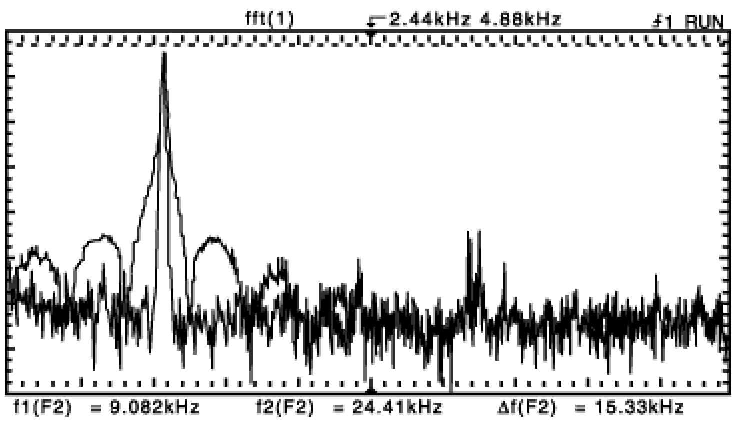
3. Does the Hanning window exhibit more or less spectral leakage when compared to the rectangular window?

Conclusions

1. The spectral resolution of the FFT is improved by lowering the effective sampling rate using the Time/Div control.

2. The effective sampling rate should be higher than the input signal's Nyquist rate to avoid aliasing.

3. The rectangular window exhibits a large degree of spectral leakage when compared to the Hanning window. The poor spectral leakage property of the rectangular window overshadows its good spectral resolution capability.



Function 2: Menu		
Effective Sampling Rate	**Units/div**	**Ref Level**
10 kSa/s	**10.00 dB**	**5.00 dBV**
FFT Menu		
Center Freq.	**Freq. Span**	**Window**
2.441 kHz	**4.883 kHz**	**Hanning, Rectangular**

Figure 2.6

The 1024 DFT (Magnitude) of a 3.5 *V* (peak-to-peak) 1 kHz sinusoid. The oscilloscope display features an overlap of the signal spectrum using both the Hanning and rectangular windows. This display was created by first selecting the settings shown in

the box below and then storing each "trace" using the *Trace* features of the oscilloscope. By turning on both traces, we obtain the display shown above.

Answers to Questions

1) Lowering the sampling rate improves spectral resolution.

2) Yes, although a lower sampling rate improves spectral resolution, the sampling rate must be high enough to prevent aliasing.

3) The Hanning window exhibits significantly less spectral leakage compared to the rectangular window.